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A qualitative study of the optimal control model for an electric power generating system

Yidiat O. Aderinto

Mathias O. Bamigbola
Department of Mathematics, University of Ilorin, Ilorin, Nigeria

Abstract
The economic independence of any nation depends largely on the supply of abundant and reliable electric power and the extension of electricity services to all towns and villages in the country. In this work, the mathematical study of an electric power generating system model was presented via optimal control theory, in an attempt to maximize the power generating output and minimize the cost of generation. The factors affecting power generation at minimum cost are operating efficiencies of generators, fuel cost and transmission losses, but the most efficient generator in the system may not guarantee minimum cost as it may be located in an area where fuel cost is high. We choose the generator capacity as our control $u_i(t)$, since we cannot neglect the operation limitation on the equipment because of its lifespan, the upper bound for $u_i(t)$ is choosing to be 1 to represent the total capability of the machine and 0 to be the lower bound. The model is analyzed, generation loss free equilibrium and stability is established, and finally applications using real life data is presented using one generator and three generator systems respectively.

Keywords: mathematical model, electric power generating system, generation loss free equilibrium

1. Introduction

'Let there be light and there was light shone forth! The world saw it and it was good! And the world was revolutionized by the light called "electricity"' (Manafa 1978).

It is no exaggeration that the whole of mankind, indeed the entire world economy, is today governed by the forces of electricity. We turn on the switch and light is made, as a result; we cook our food with an electric cooker, heat our room with an electric heater and cool them with an air conditioner, listen to radio, watch television, fly a rocket and jet to the moon and other planets, speak to distant friends and relations by means of telephone and the radio, and indeed, enjoy many amenities. Behind these, electricity is at work.

Several authors have worked on the application of optimal control including numerical application. (Fister et al., 1998) worked on optimizing chemotherapy in an HIV model, (Fister and Panetta 2000) worked on optimal control applied to cell–cycle specific cancer chemotherapy, (Burden et al., 2003) considered optimal control applied to immunotherapy, and (Agusto, 2008) worked on optimal control of oxygen absorption in aquatic systems. Others whose research touched on application of optimal control include (Bao-Zhu and Tao-Tao 2009), (Erika et al., 2007), (Kathirgamanathan and Neitzart 2008), (Kirshner 1996), (Salley 2007), to mention few. In addition, several researchers have also worked on the electric power system. These include (Lee et al., 1988), (Billinton 1994), (Branimir et al. 1993), (Shaidehpour et al., 1988), (Ehsani et al., (1968) to name a few. As such, much emphasis has been on the operational (design) aspect rather than the economical aspect of an optimal power flow problem of electric power generation.

The purpose of this work therefore, is to qualitatively study a mathematical model in the form of an optimal control model, (Aderinto and Bamigbola 2010) for a better understanding of electric power generation, in an attempt to minimize the cost of generation, and maximize the generator output without violating operating limitations on the equipment.

2. A mathematical model of an electric power generating system

Let $G_i(t)$ represent the amount of power generated by the i^{th} generator at time t, and $C_i(t)$ the capital investment on the the i^{th} generator at time t. For the control classes, we choose measurable functions defined on a fixed interval, $a_j \leq u_i \leq b_j$ $(i = 1 \ldots,$

m), since we cannot neglect the physical law governing power generating systems and the operating limitation of the equipment.

Considering the ith generator, the rate of change in generation at time (t) depends on invested capital $C_i(t)$ and the generator output $G_i(t)$ which in turn, depends on the power input, generator capacity, running cost and transmission losses. Suppose we have m generators, (i.e., $i = 1,2,\ldots,m$) then, we have:

$$\frac{dG_i(t)}{dt} = \alpha_i + q_i C_i(t) G_i(t) - k_i G_i(t),$$

$$i = 1,2,\ldots,m \qquad (2.1)$$

where α_i, q_i and k_i are respectively the actual mechanical/electrical energy from the high pressure turbine and low pressure turbine (capacity of generator i), the corresponding running cost and the transmission loss rate, which depends on the distance from the grid centre. Also, the investment on capital $C_i(t)$ at time (t) is known to be dependent on labour cost s_i, maintenance cost y_i, fuel cost $r_i C_i(t) G_i(t)$, capacity rate x_i, and the cost of transmission to the grid centre $\gamma_i C(t)$, because of the physical law that governs power generation and the operating limitation on the equipment, we choose the generator capacity as our control $u_i(t)$, since we cannot neglect the operation limitation on the equipment because of its lifespan, the upper bound for $u_i(t)$ is choosing to be 1, to represent the total capability of the machine . Thus, we have:

$$\frac{dC_i(t)}{dt} = (s_i + y_i) + r_i C_i(t) G_i(t) - x_i u_i(t) C_i(t) + \gamma_i C_i(t), \; i = 1,2,\ldots,m \qquad (2.2)$$

In the above setting an important objective is to minimize the total operating cost incurred in the process of generating the required quantity of electric power G at any time t, and the components of the total operating cost are $C(t)$ and u (t). Thus, the expression for the objective function is of the form:

$$J(u) = \int^{t_f} \left[\delta^T C(t) + \eta u^T u \right] dt,$$

Where $\delta = (\delta_1, \delta_2, \ldots, \delta_m)$ is the unit expenditure on the generators, η is a parameter to balance the size of the control.

The problem to study is to find the control u that minimizes the cost function:

$$J(u) = \int^{t_f} \left[\delta^T C(t) + \eta u^T u \right] dt,$$

subject to:

$$\frac{dG_i(t)}{dt} = \alpha_i + q_i C_i(t) A G_i(t) - k_i G_i(t)$$

$$\frac{dC_i(t)}{dt} = (s_i + y_i) - C_i(t) D G_i(t) - u^T(t) E C_i(t) + y_i C(t), \; G_i(t_0) = G_0, \; C_i(t_0) = C_0, \; a_i \leq u_i \leq b_i$$

3. Generation loss free equilibrium and stability

A good strategy to achieve the objective of attaining maximum power output at minimum cost is to minimize the electric power generation losses. In this connection, we determine the equilibrium point for the system and establish that the system is both stable and generation loss free at this point.

Definition 3.1 Equilibrium point

Let us consider the system:

$$\frac{dx_1}{dt} = P(x_1, x_2), \text{ and dx2 dt} = Q(x_1, x_2)$$

A point (x_1^0, x_2^0) for which $P(x_1^0, x_2^0) = 0 = Q(x_1^0, x_2^0)$ is called an equilibrium point or a critical point of the system. The point (x_1^0, x_2^0) is a trajectory point, i.e., the solution stating at this point, always remains within reasonable distance of it. The equilibrium point according to (Shabi and Abo-Zeid, 2010) is called locally asymptotically stable if it is locally stable, global attractor i.e., if every solution converges to that point as $n \to \infty$, and globally asymptotically stable if it is locally asymptotically stable and global attractor. According to (Cao and Wang, 2003), the equilibrium point $x^* = (x^*_1, x^*_2, \ldots, x^*_n)$ is said to be globally asymptotically stable if it is locally stable in the sense of Lyapunov and global attractive, where global attractivity means that every trajectory tends to x^* as $t \to +\infty$.

The global asymptotic stability of an equilibrium point of a differential system can be expressed according to an elementary result in stability theory which stated that if the jacobian matrix of function f, i.e., $Jf(x)$, has eigenvalues with negative real part at a singular point, then the point is asymptotically stable. In other words if $Jf(x)$ has eigenvalues with negative real part at any critical point in IR , then the critical point is globally asymptotically stable, (Sabatini, 1990).

Definition 3.2 Generation loss-free equilibrium

The generation loss-free equilibrium (GLFE) of the model is obtained by setting the right hand side of the equation to zero and taking all the generator output and production cost terms in the equations to be zero. Thus, there is a steady state (equilibrium point) of the system called generation loss free equilibrium, i.e., a state where there is no generation loss as t tends to infinity (after a long term has passed). For more on free equilibrium see (Bhunnu *et al.*, 2008) and (Castilio-Chavez *et al.*, 2007)

Definition 3.3 Stability of a system

A system is said to be locally stable if its weight function response decays to zero as t tends to infinity. The system is asymptotically stable if and only if the zero of the characterization function $s^n = a_1 s^{n-1} = \dots = a_n = 0$ i.e., the finite poles of the transfer function are negative for the real zeros or have negative real parts (for complex zeros). In other words, a system is asymptotically stable if λ_i is negative where λ_i are the eigenvalues.

On the other hand, if each zeros is 1, the system is marginally stable but if its greater than 1, then the system is unstable, (Craven, 1995); (Burghes and Graham 1989).

The generation loss-free equilibrium (GLFE) of the model is obtained as follows, using the Definitions, we obtain the following equations:

$$0 = \alpha_i + q_i C_i(t) G_i(t) - k_i G_i(t), \quad i = 1,2,\dots,m \tag{3.1a}$$

$$0 = (s_i + y_i) + r_i C_i(t) G_i(t) - x_i u_i(t) C_i(t) + \gamma_i C_i(t), \quad i = 1,2,\dots,m \tag{3.1b}$$

At the equilibrium points $(\hat{G}_i, \hat{C}_i)^T$, equation (3.1a and 3.1b) becomes:

$$0 = \alpha_i + q_i \hat{C}_i(t) \hat{G}_i(t) - k_i \hat{G}_i(t), \quad i = 1,2,\dots,m \tag{3.2a}$$

$$0 = (s_i + y_i) + r_i \hat{C}_i(t) \hat{G}_i(t) - x_i u_i(t) \hat{C}_i(t) + \gamma_i \hat{C}_i(t), \quad i = 1,2,\dots,m \tag{3.2b}$$

The system is said to be stable if all the eigenvalues of the system are negative.

We now state and prove the following theorems for the local stability of the generation loss-free equilibrium at $(\hat{G}_i, \hat{C}_i)^T$.

Theorem 3.1

The GLFE is asymptotically stable when the basic loss production number $\lambda_i = L_0 < 1$ and unstable for $\lambda_i = L_0 > 1$

Proof:

To study the stability of different equilibrium points, we have to determine the Jacobian matrices around the points. Considering the Jacobian of the matrix at the equilibrium point:

$$E = (\hat{G}_i, \hat{C}_i)^T = \left(\frac{x_i u_i \hat{C}_i - \gamma_i \hat{C}_i - (s_i + y_i)}{r_i \hat{C}_i}, \frac{k_i \hat{G}_i - \alpha_i}{q_i \hat{G}_i} \right)^T$$

given by

$$J(\hat{G}_i, \hat{C}_i) = \begin{pmatrix} q_i \hat{C}_i(t) - k_i & q_i \hat{G}_i(t) \\ r_i \hat{C}_i(t) & -x_i u_i + r_i \hat{G}_i(t) + \gamma_i \end{pmatrix} = \begin{pmatrix} q_i \hat{C}_i - k_i & q_i \hat{G}_i \\ r_i \hat{C}_i & -x_i u_i + r_i \hat{G}_i + \gamma_i \end{pmatrix} \tag{3.3}$$

Evaluating the Jacobian at the equilibrium point E,we obtain:

$$J(E) = \begin{pmatrix} q_i \dfrac{k_i \hat{G}_i - \alpha_i}{q_i \hat{G}_i} - k_i & q_i \dfrac{x_i u_i \hat{C}_i - \gamma_i \hat{C}_i - (s_i + y_i)}{r_i \hat{C}_i} \\ r_i \dfrac{k_i \hat{G}_i - \alpha_i}{q_i \hat{G}_i} & -x_i u_i + r_i \dfrac{x_i u_i \hat{C}_i - \gamma_i \hat{C}_i - (s_i + y_i)}{r_i \hat{C}_i} + \gamma_i \end{pmatrix}$$

Finding the determinant of the characteristics of the jacobian matrix at E, we have:

$$Det[J(E)] = \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0, \text{ where}$$

$$a_{11} = q_i \frac{k_i \hat{G}_i - \alpha_i}{q_i \hat{G}_i} - k_i,$$

$$a_{12} = q_i \frac{x_i u_i \hat{C}_i - \gamma_i \hat{C}_i - (s_i + y_i)}{r_i \hat{C}_i},$$

$$a_{21} = r_i \frac{k_i \hat{G}_i - \alpha_i}{q_i \hat{G}_i},$$

$$a_{22} = -x_i u_i + r_i \frac{x_i u_i \hat{C}_i - \gamma_i \hat{C}_i - (s_i + y_i)}{r_i \hat{C}_i} + \gamma_i$$

$$\Rightarrow (a_{11} - \lambda)(a_{22} - \lambda) - a_{21} a_{12},$$

$$\text{i.e., } \lambda^2 - (a_{11} + a_{22})\lambda + a_{11} a_{22} - a_{21} a_{12}$$

which can be written as:

$$\lambda^2 - p\lambda + q = 0$$

where $p = a_{11} + a_{22}$, $q = a_{11} a_{22} - a_{21} a_{12}$

The eigenvalues are:

$$\lambda = \frac{p \pm \sqrt{p^2 - 4q}}{2},$$

$$\text{i.e., } \lambda_1 = \frac{p - \sqrt{p^2 - 4q}}{2}$$

$$\text{and } \lambda_2 = \frac{p + \sqrt{p^2 - 4q}}{2} \quad (3.4)$$

Thus, the values of λ_1 and λ_2 determine the stability of the loss-free equilibrium. If the *Det[J(E)]<0* then the basic loss Production number, Lo < 1.

This implies that the generation loss-free equilibrium point E is asymptotically stable whenever $L_0 < 1$, (i.e. when all the eigen values are negative the condition holds). For the proof of a similar result see (Bhunnu *et al.,* 2008), and (Castilio-Chavez et *al.,* 2007).

By application of the real life data (as we have on Table 4.1) we obtained λ_1 and λ_2 to be 0.0229739 and -0.0229739 respectively.

To obtain the next theorem, we utilize the following assumption by (Cao and Wang 2003).

Assumption 3.1*:* If f_i and g_i $(i = 1,2...,n)$ are Lipschitz continuous, then there exist positive constants k_i, l_i such that:

$$[f_i(u) - f_i(v)[\le k_i| u - v| ,| g_i(u) - g_i(v)| \le l_i| u - v| ,$$

for all $u, v \in \Re$ and $i = 1,2,...,n,$

Theorem 3.2

Given that Assumption (3.1) is satisfied, then equations (3.2a) and (3.2b) has a unique equilibrium point.

Proof

Let $E_1 = (\hat{G}, \hat{C})^T$ and $E_2 = (\hat{Q}, \hat{D})^T$ denote the two equilibrium points of the system model (3.2a) and (3.2b) where:

$$\hat{G} = (\hat{G}_1, \hat{G}_2, \ldots, \hat{G}_n)^T, \quad \hat{Q} = (\hat{Q}_1, \hat{Q}_2, \ldots, \hat{Q}_n)^T$$

$$\hat{C} = (\hat{C}_1, \hat{C}_2, \ldots, \hat{C}_n)^T, \text{ and } \hat{D} = (\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_n)^T$$

Then we have:

$$\alpha_i + q_i \hat{C}_i \hat{G}_i - k_i \hat{G}_i = 0$$

$$\alpha_i + q_i \hat{C}_i \hat{Q}_i - k_i \hat{Q}_i = 0 \quad (3.5a)$$

and

$$(s_i + \gamma_i) - x_i u_i \hat{C}_i + r_i \hat{C}_i \hat{G}_i + \gamma_i \hat{C}_i = 0$$

$$(s_i + \gamma_i) - x_i u_i \hat{D}_i + r_i \hat{D}_i \hat{G}_i + \gamma_i \hat{D}_i = 0 \quad (3.5b)$$

These imply that:

$$q_i \hat{C}_i (\hat{G}_i - \hat{Q}_i) = k_i (\hat{G}_i - \hat{Q}_i), \text{ and}$$

$$x_i u_i (\hat{C}_i - \hat{D}_i) = r_i \hat{G}_i (\hat{C}_i - \hat{D}_i) + \gamma_i (\hat{C}_i - \hat{D}_i), \quad i = 1, \ldots, n$$

Using the assumption above, we obtain:

$$q_i \hat{G}_i |\hat{G}_i - \hat{Q}_i| \le l_i |k_i| |\hat{G}_i - \hat{Q}_i| \quad (3.6a)$$

and

$$x_i u_i |\hat{C}_i - \hat{D}_i| \le \beta_i |r_i \hat{G}_i| |\hat{C}_i - \hat{D}_i| + l_i |\gamma_i| |\hat{C}_i - \hat{D}_i| \quad (3.6b)$$

Rewriting equation (3.6a) and (3.6b) respectively as:

$$(A - |K|L)(|\hat{G}_1 - \hat{Q}_1|, |\hat{G}_2 - \hat{Q}_2|, \ldots, |\hat{G}_n - \hat{Q}_n|)^T \le (0, 0, \ldots, 0)^T \quad (3.7a)$$

and

$$(A - |R|\beta - |\gamma|L)(|\hat{C}_1 - \hat{D}_1|, |\hat{C}_2 - \hat{D}_2|, \ldots, |\hat{C}_n - \hat{D}_n|)^T < (0, 0, \ldots, 0)^T \quad (3.7b)$$

Multiplying both sides of (3.7a) by $(A - |K| L)^{-1}$ and both sides of (3.7b) by $(A - |R| \beta - |\gamma| L)^{-1}$ we obtain:

$$(|\hat{G}_1 - \hat{Q}_1|, |\hat{G}_2 - \hat{Q}_2|, \ldots, |\hat{G}_n - \hat{Q}_n|) \le (0, 0, \ldots, 0)^T$$

and

$$(|\hat{C}_1 - \hat{D}_1|, |\hat{C}_2 - \hat{D}_2|, \ldots, |\hat{C}_n - \hat{D}_n|)^T \le (0, 0, \ldots, 0)^T$$

which implies that for all $i = 1,2,...n$,

$$\hat{G}_i = \hat{Q}_i \quad \text{and} \quad \hat{C}_i = \hat{D}_i$$

Hence, our model system has a unique equilibrium point.

4. Application to real life data

Electric power generation as a real life endeavour needs to be studied as a multidisciplinary subject making use of contributions from the relevant fields. As shown in the proceeding developments in this work, mathematics has a lot to contribute in resolving practical problems in electric power generation as well as improving on its advancement. In what follows, we fashioned solutions to the electric power generating system model using real life data.

The following tabulated values were obtained from the National Control Centre, Osogbo, Nigeria.

Table 4.1: Generator parameters for a one-generator station (National Grid Centre, Osogbo)

Parameter	*Meaning*	*Value*
α	actual mechanical/electrical energy available from the turbine	800MW
q	total running cost	0.3217 per unit
r	fuel cost rate	0.347 per unit
x	actual capacity rate	0.606
k	rate of energy loss during transmission	0.002 MW per unit
s	labour cost	200 per h (assumed)
y	maintenance cost	#100 per h (assumed)
γ	Cost of transmitting from generating station	# 0.3421 per unit
u	generator actual capacity rate (control)	$a \leq u \leq b$, $0 \leq u \leq 1$, $a = 0$, $b = 1$
δ	unit of power generating station	1, 2
η_1	Parameters to balance the size of the control. (Number of hours for which the machines can be on)	2, 6

Table 4.2: Generator parameters for a three-generator station (National Grid Centre, Osogbo, Nigeria)

Parameter	*Meaning*	*Value*
a	actual mechanical / electrical energy available from the turbine	100MW,100MW,80MW G_1 100MW, G_2 100MW, $G_{3\ 8}$80MW,
q	total running cost	0.3217, #0.3112, 0.312Per unit
r	fuel cost rate	0.3478 each per unit
x	actual capacity rate	0.606, 0.502, 0.402
k	rate of energy loss during transmission	0.002 MW each per unit
s	labour cost	70 per h (assumed)
y	maintenance cost	50 per h (assumed)
γ	Cost of transmitting from generating station	0.3421 per unit
u	generator actual capacity rate (control)	$a \leq u \leq b$, $0.3 \leq u \leq 0.9$, $a = 0.0$, $b = 1.0$
δ	unit of power generating station	1 each
η	Parameters to balance the size of the control. (Number of hours for which the machines can be on)	3

4.1 Solution to the electric power generating model

In this work the desired solution is that power output from the generating station is maximized with minimum cost of production. The first variable α, is best described by the actual mechanical / electrical energy from the turbine. The second factor x i.e., the rate of generation is associated with the capacity of the generating machine. The third variable η is the number of hours for which the generating machine is going to be on.

There are two systems of differential equations in the optimality system with one involving the control. The systems is solved using an analytical method, (Matilde, 2009); (Otarod, 2008); (Pope *et al.,* 1998); (Shepley, 1966) and (Weisstein, 2010), and iterative method with fourth-order Runge–Kutta scheme, (Jain, 1983); (Hosking *et al.,* 1996) and (Eric, 2003); (Pingping 2009); (Naevadal 2003). The controls are updated at the end of each iteration using the formula for optimal controls.

The problem under consideration is:

Minimize

$$J(u) = \int_{t_0}^{t_f} \left[\delta^T C(t) + \eta u^T u \right] dt, \tag{4.1}$$

subject to:

$$\frac{dG_i(t)}{dt} = \alpha_i + C_i(t) q_i G_i(t) - k_i G_i(t) \tag{4.2}$$

$$\frac{dC_i(t)}{dt} = (s_i + y_i) + C_i(t) r_i G_i(t) - u_i(t) x_i C_i(t) + \gamma_i C(t) \tag{4.3}$$

With

$$G_i(t_0) = G_0, \quad C_i(t_0) = C_0, \quad a_i \leq u_i \leq b_i$$

By the Langrangian we have:

$$L(G, C, u, \lambda_1, \lambda_2, M, N) =$$
$$\left[\delta^T C_i + \eta u_i^T u_i\right] + \lambda_1\left[\alpha_i + C_i(t) q_i G_i(t) - k_i G_i(t)\right]$$
$$+ \lambda_{2i}\left[(s_i + y_i) - x_i u_i C_i(t) + r_i C_i(t) G_i(t) + \gamma_i C(t)\right]$$
$$+ \sum_{i=1}^{m} M_i(b_i - u_i) + N_i(u_i - a_i).$$

where $M_i,\ldots,M_m, N_i,\ldots N_m \geq 0$ are penalty numbers satisfying

$M_i (b_i - u_i) = 0, N_i(u_1 - a_1) = 0$, *at* u_i^*

Thus, using the data in Tables 4.1 and 4.2, the computations for the numerical and analytical solution are obtained respectively as follows.

Table 4.4: Analytical solution for one-generator electric power model

$h = 0.1$, $u = 0.1, 0.2, 0.3, \ldots, 1.0$, $\delta = 1$, $\eta = 6$, G = 799.205		
U	C	J
0.1	312.817575	390.035551
0.2	322.625669	390.331445
0.3	332.843022	390.870626
0.4	343.488833	391.653101
0.5	354.583257	392.67888
0.6	366.147464	393.94797
0.7	378.203684	395.460387
0.8	390.775265	397.216131
0.9	403.886734	399.215214
1.0	403.926917	400.986917

Table 4.3: Numerical solution for one-generator electric power model

$h = 0.05$, $u = 0.1, 0.2, 0.3, \ldots, 1.0$, G = 782.65698				
U	C	J $t_f = 1$	J $t_f = 1 = 4$	J $t_f = 4$
		$\delta = 1$, $\eta = 6$	$\delta = 2$, $\eta = 6$	$\delta = 2$, $\eta = 6$
0.1	383.975551	384.035551	760.010902	3040.043608
0.2	385.231445	385.471445	770.702890	3082.81156
0.3	386.490626	387.030626	773.521252	3094.085008
0.4	387.7531012	388.713101	776.466202	3105.864808
0.5	389.018880	390.518880	779.537760	3118.15104
0.6	390.287973	392.447973	782.735416	3130.941664
0.7	391.560387	394.500387	786.060774	3144.243096
0.8	392.836131	396.676131	789.512262	3158.049048
0.9	394.115214	398.975214	793.090428	3172.361712
1.0	394.926917	400.926917	795.853834	3183.415336

Table 4.5: Numerical solution for a three-generators electric power model

$h = 0.05$, $u = 0.2, 0.3, 0.4, \ldots, 0.9$, $\delta = 1$, $\eta = 3$, $i = 1, 2, 3$					
u	C_1	C_2	C_3	*J at* $t_f = 1$	J at $t_f = 6$
0.2	186.95889	186.63370	186.32130	560.27389	3361.64336
0.3	187.90813	187.56171	186.38327	562.66311	3375.97864
0.4	188.86003	188.20629	187.57888	566.08521	3396.51124
0.5	189.81461	189.33645	188.20943	569.63048	3417.78290
0.6	190.63340	189.78621	188.84115	572.50077	3435.00463
0.7	191.73181	190.57893	189.47405	576.19479	3457.16875
0.8	191.91594	191.37349	190.10813	579.15756	3474.94538
0.9	193.11363	192.16990	190.74339	583.23692	3499.42155
G1	99.900				
G2		99.850			
G3			77.920		

Table 4.6: Numerical solution for a three-generators electric power model

$h = 0.05$, $u = 0.3, 0.4, ..., 0.9$, $\delta = 1$, h = 3, i = 1, 2, 3, C_1, C_2, C_3 in Table 2					
u	u_1	u_2	u_3	J at $t_f = 1$	J at $t_f = 6$
0.2	0.3	0.4	0.5	565.82385	3394.94311
0.3	0.4	0.5	0.6	567.03764	3402.22584
0.4	0.5	0.6	0.7	572.37487	3434.24923
0.5	0.6	0.7	0.8	575.79047	3454.74280
0.6	0.7	0.8	0.9	579.66869	3478.01215
0.7	0.8	0.3	0.7	572.61170	3435.67021
0.8	0.9	0.8	0.5	577.79656	3466.77933
0.9	0.8	0.6	0.8	576.73028	3460.38170
	0.9	0.9	0.5	579.10297	3474.61780
	0.8	0.7	0.5	574.84430	3449.06581

4.2. Discussion of results

From these tables, it can be established that the model gave the maximum generator output so far, and that the more we generate the more we spend on it. Therefore, the choice of u_i is greatly dependent on the number of generating machines that are available and the number of hours or the duration in which the generation is to be carried out. Thus $u_1 = 0.8$, $u_2 = 0.7$, and $u_3 = 0.5$ is recommended for the three generators system above with $J = 574.84430$ at $t_f = 1$ and $J = 3449.06581$ at $t_f = 6$.

However, the physical capability of the machines or simply the physical characteristics of the generating machines is very important and so the control has to be put into consideration while trying to minimize the cost. As such, we can monitor the control i.e., we can generate more with minimum cost and still maintain the good condition of the generating machine. It was also observed that the more time we spend, the more power and the more cost we have. Thus, from the results obtained, it is observed that for efficiency and effective functioning of the generating machines in each station, monitoring of the control is very essential.

5. Conclusion

Electrical engineers are concerned with the technology of generation, transmission, distribution, and utilization of electric energy. Since electric energy systems is probably the largest and most complex industry in the world, the electrical engineers offer some challenging problems in designing future power system to deliver sufficient electrical energy in a safe, clean, ecological, and economical manner. Hence, the need to improve the quality and quantity of electric power generation is done by applying optimal control theory to the study of electric power generation. To a layman, the result can be interpreted by saying that, the electric power generating systems can be expressed mathematically by using mathematical equations which relates two or more parameters that can be used to measure the condition or state of electric power generating systems. These parameters enable us to know the condition and the capacity of the generator, how to use, and how long to use, so as to maximize the generator output and minimize the cost of generation.

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