Supplementary file for:

The impact of political objectives on optimal electricity generation and transmission in the Southern African Power Pool

Philipp A. Trotter, Roy Maconachie, Marcelle C. McManus

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Modelling details and implementation

All-pairs shortest path model formulation

The APSP problem is a closed model formulation to solve all possible instances of the well-documented simple shortest path problem (Dijkstra, 1959; Dreyfus, 1969). It finds the shortest path between each supply node in supply country $s \in S$ of generation technology $g \in G$ and demand node $d \in D$. This paper assumes that the shortest distance maps linearly to the minimum transmission costs between a supply and demand node. The APSP can be solved with efficient approaches such as the Floyd-Warshall algorithm which is useful for large problem instances (Floyd, 1962). Here, the model size allows a straight-forward closed model formulation of the APSP problem, which is presented below.

Let set ST denote the set of all supply nodes (s,g) where the energy potential is greater than 0. Let $N = ST \cup D$, where N is the set of all combined potential supply nodes ST and demand nodes D. The shortest path model guarantees that decision variable $xBin_{abr_ar_b}$ is always exactly 1 if the shortest path between node $a \in N$ and node $b \in N$ contains the link from node $r_a \in N$ to $r_b \in N$, and 0 otherwise. This is the case even if $xBin_{abr_ar_b}$ it is defined to be continuous between 0 and 1(Dijkstra, 1959), yielding a linear programming (LP) model with fast solution times.

The objective function (SM.1) minimises the total distance covered when sequentially travelling from each node $a \in N$ to each node $b \in N$. This yields an equivalent result to solving each shortest path problem between each node separately.

$$\min \sum_{a \epsilon N} \sum_{b \epsilon N} \sum_{r_a \epsilon N} \sum_{r_b \epsilon N} xBin_{abr_ar_b} \cdot distance_{r_ar_b} \qquad \text{(SM.1)}$$

Parameter $distance_{r_ar_b}$ denotes the distance between nodes $r_a \in N$ and $r_b \in N$.

The set of constraints (SM.2) – (SM.7) are a straight-forward extension of the simple shortest path problem to cover all node connections $n \in N$ to $m \in N$. Parameter $neighbour_{abr_ar_b}$ is 1 if node r_b can be reached from node r_a without crossing into a third country and if r_b is not a supply node, and 0 otherwise.

$$\sum_{r_b \in N} xBin_{abar_b} - \sum_{r_b \in N} xBin_{abr_ba} = 1 \qquad \forall a \in N, b \in N, a \neq b \qquad \text{(SM.2)}$$

$$\sum_{r_b \in N} xBin_{abr_ar_b} - \sum_{r_b \in N} xBin_{abr_br_a} = 0 \qquad \forall a \in N, b \in N, r_a \in N; r_a \neq a \wedge r_a \neq b \qquad \text{(SM.3)}$$

$$\sum_{r_b \in N} xBin_{abbr_b} - \sum_{r_b \in N} xBin_{abr_bb} = -1 \qquad \forall a \in N, b \in N, a \neq b \qquad \text{(SM.4)}$$

$$xBin_{abr_ar_b} = 1 \qquad \forall a \in N, b \in N, r_a \in N, r_b \in N; r_b = a = b = r_a \qquad \text{(SM.5)}$$

$$xBin_{abr_ar_b} \leq neighbour_{r_ar_b} \qquad \forall a \in N, b \in N, r_a \in N, r_b \in N \qquad \text{(SM.6)}$$

$$0 \leq xBin_{abr_ar_b} \leq 1 \qquad \forall a \in N, b \in N, r_a \in N, r_b \in N \qquad \text{(SM.7)}$$

Problem APSP, (SM.1) – (SM.7), has an advantage and a disadvantage over solving all shortest path problems separately. Its advantage is that it requires only one model initialisation and one output read operation to yield all possible shortest paths in the network. The instance of this model discussed in section 4 of this paper has 12 demand nodes and 41 supply nodes. Separately solving the relevant shortest path problems would require $12 \cdot 41 = 492$ iterations, each with a separate model initialisation and output read, a number that increases exponentially with larger model instances. The disadvantage is that model APSP yields a range of shortest paths that are not required for the later multi-criteria planning optimisation. In addition to solving the shortest path between all supply and demand nodes, the model also solves the shortest path between any two supply, and any two demand nodes, neither having any relevance for the planning optimisation. However, due to the linearity of the model, a standard desktop computer using IBM ILOG CPLEX 12.7 was able to solve the problem for 12 supply and 41 demand nodes in less than 30 seconds computational time, yielding an overall solution time which is highly likely to be smaller compared to initialising and solving 492 models separately. This computational time would likely be even shorter if advanced algorithms like current Floyd-Warshall approaches would have been implemented.

The minimum transmission cost between any supply node in country s of generation technology g and demand node d can now be calculated linearly as shown in (SM.8). Decision variable $xBin_{sdr_ar_b}$ also indicates the geo-referenced path between each supply and demand node. In expression (SM.8), $unitTranscost_g$ is the levelised per kilometre and GWh transmission cost for generation technology g (see section 3). Parameter $lossCost_g$ is the unit cost of the transmission losses occurring when transmitting electricity generated by technology g (see section 3).

$$minTranscost_{sgd} = unitTranscost_g \sum_{r_a \in N} \sum_{r_a \in N} xBin_{sdr_ar_b} \cdot distance_{r_ar_b} + lossCost_g \sum_{r_a \in N} \sum_{r_a \in N} xBin_{sdr_ar_b} \cdot distance_{r_ar_b} \quad \forall s \in S, d \in D, t \in T \tag{SM.8}$$

Multi-objective linear programming (MOLP) electricity planning optimisation implementation

Due to the linearity of the problem, any one of the three objectives, namely cost minimisation, GHG emission minimisation and national electricity sovereignty maximisation, can be modelled as an objective function and the remaining two as variable constraints. The model is arguably most intuitive when costs are modelled as an objective function, GHG emissions and electricity sovereignty are introduced as constraints. The latter feature variable threshold values between 0 and 100. In each model iteration, they are fixed to a certain value to solve a simple Linear Programming (LP) problem, and are then varied before the next iteration to yield the complete Pareto-optimal trade-off. This procedure is then repeated for every k of interest where $PolRisk_s < k$ to address the forth, discrete objective of decreasing political risk in the network. Section 2.3 provides more detail on the solution procedure.

Expression (SM.9) minimises the cumulative levelised system costs for all new demand between $t_1 \in T$ and termination period $t_{ter} \in T$. It sums the levelised cost of electrification (LCOE) for all new annually generated and transmitted electricity in each time period. Decision variable $elec_{sgdt}$ denotes the electricity in GWh sent from supply country s using generation technology g to demand country d in time period t.

$$\min \sum_{s \epsilon S} \sum_{g \epsilon G} \sum_{t=t_1}^{t_{ter}} \left((t_{ter} - t) gencost_{sgt} \sum_{d \in D} \left(elec_{sgdt} - elec_{sgdt-1} \right) \right) + \sum_{s \epsilon S} \sum_{g \epsilon G} \sum_{d \epsilon D} \sum_{t \epsilon T} minTranscost_{sgd} elec_{sgdt} \tag{SM.9}$$

Parameter $gencost_{sgt}$ denotes the levelised unit generation cost in supply country s by technology g at time t (in 2010 USD/GWh). Parameter $minTranscost_{sgd}$ is taken from the APSP problem solution (section 2.1) for the current level of k where $PolRisk_s < k \ \forall \ s \epsilon S$. The following constraints complete the MOLP.

$$\sum_{s\epsilon S}\sum_{g\epsilon G} elec_{sgdt} \geq demand_{dt} \qquad \forall d \in D, t \in T \qquad \text{(SM.10)}$$

$$\sum_{d\epsilon D} elec_{sgdt} \leq supply_{sg} \qquad \forall s \in S, g \in G, t \in T \qquad \text{(SM.11)}$$

$$\sum_{d\epsilon D} elec_{sgdt} \leq \sum_{d\epsilon D} elec_{sgd(t+1)} \qquad \forall s \in S, t \in T\backslash\{2030\}, g \in G \qquad \text{(SM.12)}$$

$$\frac{\sum_{s\epsilon S, s=d}\sum_{g\epsilon G}\left(elec_{sgdt}\right)}{demand_{dt}} \geq minPolSov \qquad \forall d \in D, t \in T \qquad \text{(SM.13)}$$

$$\frac{\sum_{s\epsilon S}\sum_{g\epsilon G}\left(elec_{sgd2030} \cdot CO_2emis_g\right)}{demand_{d2030} \cdot CO_2emis_{coal}} \leq maxCO_2emis \qquad \forall d \in D \qquad \text{(SM.14)}$$

$$elec_{sgdt} \in R_{\geq 0} \qquad \forall s \in S, d \in D, g \in G, t \in T \qquad \text{(SM.15)}$$

Parameter $demand_{dt}$ indicates additional electricity demand for country d between the baseline year t_0 and time t (in GWh). Parameter $supply_{sg}$ denotes the maximum generation potential from a supply node in country s and using generation technology g (in GWh) in addition to what has been installed in 2010. Constraint (2.4) is necessary to enforce the assumption of unit LCOE cost figures, requiring capacity built in previous years to be used throughout its lifetime. Constraints (SM.13) and (SM.14) impose a minimum level of national electricity sovereignty, $minPolSov$, and a maximum level of CO_2 emissions, $maxCO_2emis$, respectively. Both threshold levels are modelled to be between 0 and 100. For the CO_2 emission constraint, this is the case because the unit CO_2 emissions of coal, CO_2emis_{coal}, are the highest of all generation technology specific unit CO_2 emissions considered here.

Solution approach

Figure A1 illustrates the solution algorithm used in this study. The optimal trade-offs between costs, GHG emissions and political electricity sovereignty at different allowed levels of supply country political risk are obtained by solving a series of two different optimisation models sequentially. First, the APSP problem is solved for three different values of allowed supply country political risk values $PolRisk_s$, namely k = 100, 80 and 60. This impacts model parameter $neighbour_{abr_ar_b}$ as shown in the supplementary material on the APSP problem. Once the APSP model is solved, the minimum transmission cost $minTranscost_{sgd}(k)$ for all k can be calculated. The solution algorithm uses this result as well as a number of further input data for its parameters to solve model MOLP, (SM.9) –

(SM.15), repeatedly along the entire range of allowed electricity sovereignty $minPolSov$ and CO_2 emissions $maxCO_2emis$ to yield a continuous optimal trade-off function. For the purpose of this study, the step change has been set to 5, leading to solving $21 \cdot 21 = 441$ LP instances of model MOLP for each of the three values for k.

All optimisation models have been implemented using the IBM ILOG CPLEX optimisation package. Due to the linearity of the models, a standard desktop computer was able to solve the (APSP) problem in under 1 minute and the multi-criteria optimisation model in under 1 hour of computational time. QGIS was used for geo-referencing and map illustrations in the study.

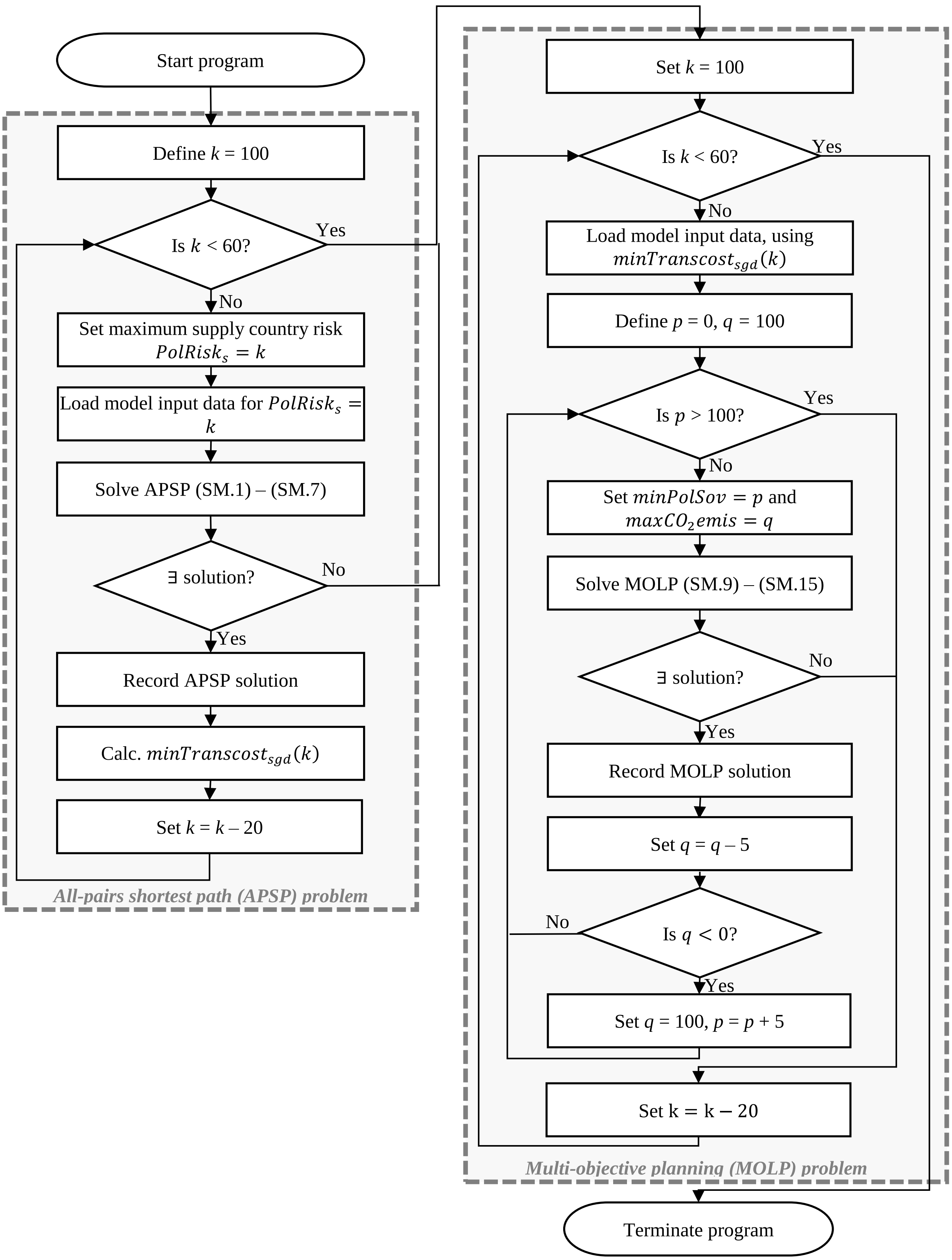

Figure A1: Schematic model solution algorithm.

Supp. material II: Data sources for all model parameters

Table A.1: Data sources of all model parameters.

Model-elements	Description	Data source
Sets		
$s\epsilon S$	12 potential electricity supply countries in the SAPP network	-
$d\epsilon D$	12 electricity demand countries in the SAPP network	-
$g\epsilon G$	6 generation technologies where primary energy is present in SAPP (solar PV, wind, hydro, geothermal, coal, oil)	-
$t\epsilon T$	20 time periods (2011, 2012, … , 2030)	-
Parameters		
$distance_{r_a r_b}$	Shortest direct distance in km between node r_a and node r_b in the network	(Natural Earth 2017)
$neighbour_{r_a r_b}$	Binary variable that is 1 if node r_a and node r_b can be connected directly without crossing through a third country, and where start node r_a is not in a country with political risk greater than *k*	(Natural Earth 2017)
k	Level of maximum supply country political risk for scenario analyses, assumed to be 100, 80 or 60 in this study	*This study*
$unitTranscost_g$	Transmission system cost per GWh and per 1000 km for each generation technology *g*	(Sanoh et al. 2014, Milligan 2012, Bahrman 2007)
$lossCost_g$	Costs of transmission losses per 1000 km transmitted for each generation technology *g*	(Sanoh et al. 2014, International Renewable Energy Agency 2013, Buys et al. 2007)
$gencost_{sgt}$	Generation cost in supply country *s* of electricity produced with generation technology *g* at time *t*	(International Renewable Energy Agency 2013)

Model-elements	Description	Data source
$transcost_{sdgt}$	Transmission cost from supply country *s* to demand country *d* of electricity produced with generation technology *g* at time *t*	(Sanoh et al. 2014, International Renewable Energy Agency 2012)
$demand_{dt}$	Electricity demand projection in MWh of country *d* at time *t*	(International Renewable Energy Agency 2013)
$supply_{sgt}$	Maximum potential supply in MWh cost in country *s* of electricity produced with generation technology *g* at time *t*	(International Renewable Energy Agency 2013, Buys et al. 2007)
$minPolSov$	Minimum required political electricity sovereignty for any given demand country, range from 0 – 100.	-
CO_2emis_g	CO_2 emissions from generation technology *g* for the production of 1 GWh electricity	*This study*
$maxCO_2emis$	Maximum allowed CO_2 emission for all countries, percentage of theoretical per country CO_2 emission maximum where all new demand between 2011 and 2030 is met through coal, range from 0 – 100.	-

Supp. material III: Optimal capacity additions for no political risk constraints for 4 different scenarios

Table A2: Numerical values of optimal capacity additions for no political risk restrictions.

Country	Scenario	Coal [GW]	Geothermal [GW]	Hydro [GW]	Oil [GW]	Solar [GW]	Wind [GW]	For export [%]
Angola	Cost min.	0	0	0	0	0	0	-
	No Import	0	0	2.73	0	0	0	0
	No CO_2	0	0	4.42	0	0	0	100
	Full restr.	0	0	2.73	0	0	0	0
Botswana	Cost min.	0	0.29	0	0	0	0	0
	No Import	0	0.29	0	0	0.76	0.29	0
	No CO_2	0	0.29	0	0	1.60	0.29	13.4
	Full restr.	0	0.29	0	0	0.76	0.29	0
DRC	Cost min.	0	0	16.79	0	0	0	65.0
	No Import	0	0	5.88	0	0	0	0
	No CO_2	0	1.08	39.09	0	0	0	85.4
	Full restr.	0	0	5.88	0	0	0	0
Lesotho	Cost min.	0	0	0.10	0	0	0.04	0
	No Import	0	0	0.10	0	0.06	0.04	0
	No CO_2	0	0	0.10	0	0.40	0.04	26.3
	Full restr.	0	0	0.10	0	0.06	0.04	0
Malawi	Cost min.	0	0	0.22	0	0	0.29	0

Country	Scenario	Coal [GW]	Geothermal [GW]	Hydro [GW]	Oil [GW]	Solar [GW]	Wind [GW]	For export [%]
	No Import	0	0	0.22	0	0.73	0.29	0
	No CO_2	0	0	0.22	0	0.06	0.29	0
	Full restr.	0	0	0.22	0	0.73	0.29	0
Mozambique	Cost min.	0	0	1.25	0	0	0	20.2
	No Import	0	0	0.99	0	0	0	0
	No CO_2	0	0	1.26	0	0	0.39	71.7
	Full restr.	0	0	0.99	0	0	0	0
Namibia	Cost min.	0	0.35	0.23	0	0	0	59.9
	No Import	0	0.34	0.23	0	0	0	0
	No CO_2	0	0.35	0.24	0	0	0.48	55.4
	Full restr.	0	0.34	0.23	0	0	0	0
South Africa	Cost min.	33.87	0.17	0.04	0	0	2.59	0.2
	No Import	33.89	0.17	0.04	0	0	2.59	0
	No CO_2	0	0.17	0.04	0	37.11	0	0
	Full restr.	0	0.17	0.04	0	120.84	0	0
Swaziland	Cost min.	0	0	0	0	0	0.01	0
	No Import	0	0	0	0	0.36	0.01	0
	No CO_2	0	0	0	0	2.33	0.01	84.0
	Full restr.	0	0	0	0	0.36	0.01	0
Tanzania	Cost min.	0	0.35	0.98	0	0	0	0

Country	Scenario	Coal [GW]	Geothermal [GW]	Hydro [GW]	Oil [GW]	Solar [GW]	Wind [GW]	For export [%]
	No Import	0	0.35	0.98	0	1.88	3.02	0
	No CO_2	0	0.35	0.98	0	0	2.27	0
	Full restr.	0	0.35	0.98	0	1.88	3.02	0
Zambia	Cost min.	0	0.23	0.91	0	0	0	100
	No Import	0	0.23	0.90	0	7.25	1.11	0
	No CO_2	0	0.23	0.91	0	0	1.11	50.6
	Full restr.	0	0.23	0.90	0	7.25	1.11	0
Zimbabwe	Cost min.	0.10	0	0.65	0	0	0	0
	No Import	1.19	0	0.65	0	0	0.31	0
	No CO_2	0	0	0.65	0	0	1.02	100
	Full restr.	0	0	0.65	0	3.27	1.01	0
SAPP total	Cost min.	33.97	1.40	21.17	0	0	0.34	22.4
	No Import	35.08	1.38	12.72	0	11.03	5.08	0
	No CO_2	0	2.48	47.91	0	41.50	8.49	45.5
	Full restr.	0	1.38	12.72	0	135.15	8.37	0

Supp. material IV: Optimal capacity additions for different scenarios

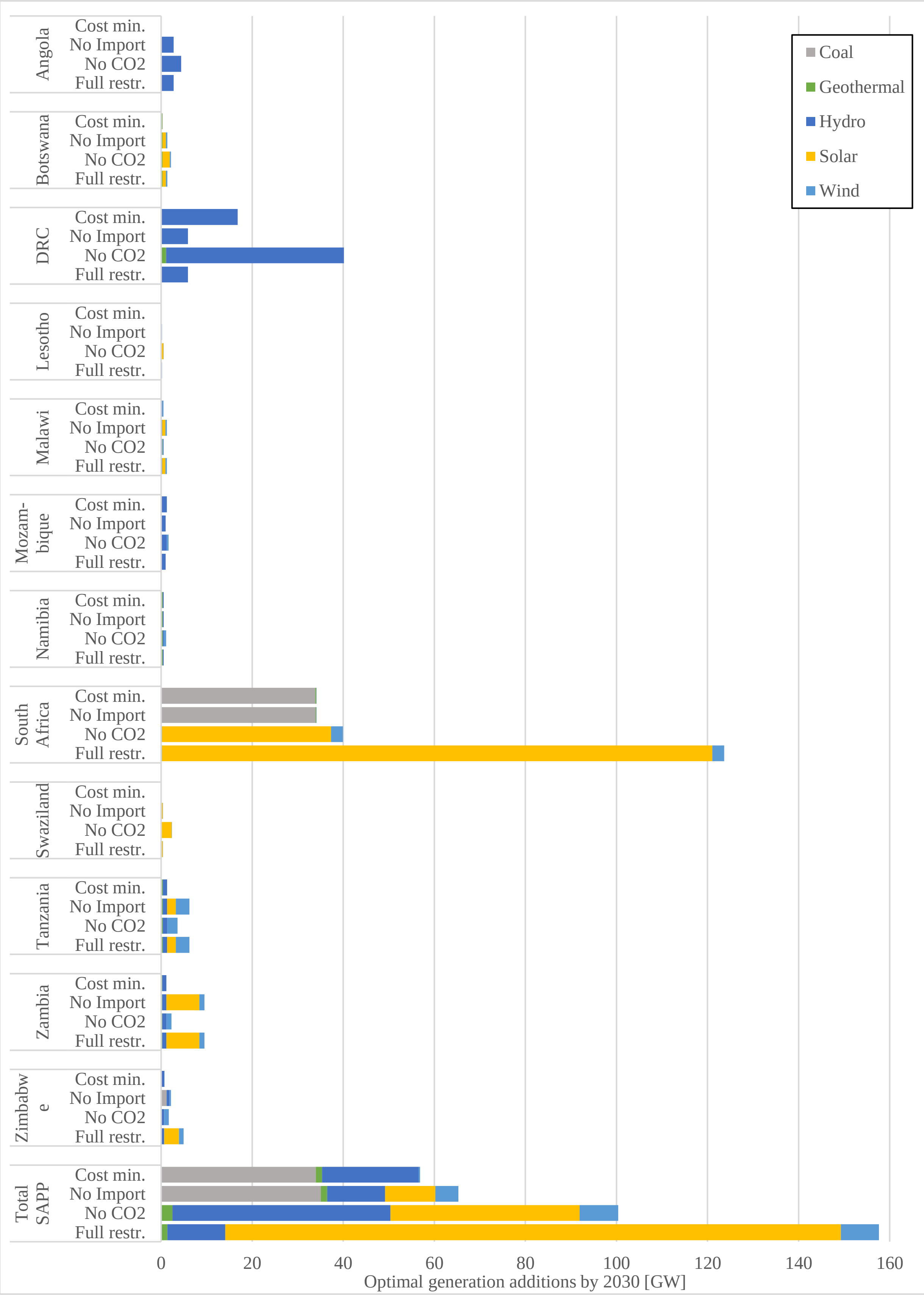

Figure 1: Illustration of optimal capacity additions for no political risk restrictions.

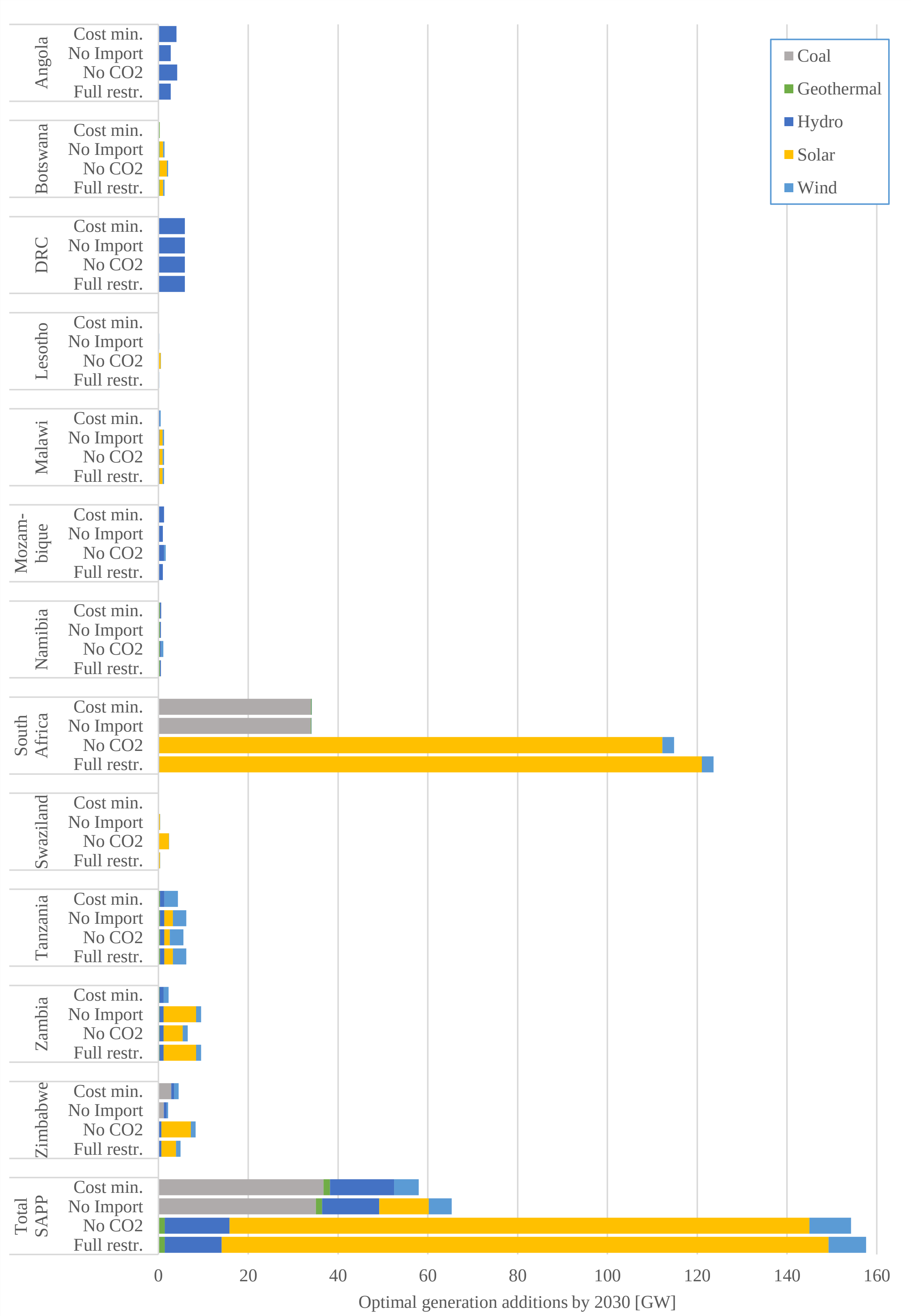

Figure 2: Illustration of optimal capacity additions for political risk restriction k < 80.